THE MINIMUM RATE OF RETURN FOR NON-LISTED COMPANIES AND ITS CONSEQUENCES

AUTHORS

- **Alfonso A. Rojo Ramírez** (Full Professor, University of Almería (Spain)
- **Salvador Cruz Rambaud** (Full Professor, University of Almería (Spain)
- **Wim Voordeckers** (Full Professor, University of Hasselt (Belgium)
- **María José Martínez Romero** (Assistant Professor, University of Almería (Spain)

Subject area: Valoración y finanzas

Key-words: Minimum rate of return, Financial risk investor, Economic risk investor, Three Components' model, Non-quoted companies

Research Workshop: 3 Información financiera y modelos de valoración

The minimum rate of return for non-listed companies and its consequences.

Abstract

This paper addresses one of the most controversial issues in the accounting and financial field: the calculus of the minimum rate of return expected by the owners-investors of the company. It differentiates between financial risk investors and economic risk investors who have different behavior against risk, demonstrating that their minimum rate of returns are different. In particular, it is shown that the minimum rate of return of the owner-investor of risk (venture investor) must be determined by the three components' model. An empirical application of this model shows the implications that professionals, researchers and scholars have to take into consideration.

1 Introduction

Determining the cost of capital remains being a central task in accounting and modern finance. Indeed, this rate helps managers to build growing strategies which support sustainable competitive advantages and also decisions about investments and executives' remuneration. Further, it is also of importance at a regional level, for example for regulators, as it is the case of the energy sector in the EU (CEER Secretariat, 2017).

Usually, its calculus is based on the one-factor CAPM (Lintner, 1965; Mossin, 1966; Sharpe, 1963) which provides an easy way to provide the minimum rate of return to managers, analysts and financial investors. Although empirical results about CAPM's goodness are poor (Fama & French, 2004) and have suffered some critiques about its capacity to measure firms' risk (Levy, 2010)(Da, Guo, & Jagannathan, 2012), it continues being the reference point to calculate the minimum rate of return to be used in investment projects, firm valuation and value creation analysis.

The CAPM, seated on the assumptions of the portfolio theory (Markowitz, 1959), considers the existence of a positive linear relationship between the expected return of securities (R_i) and the risk, represented by the market beta (β_i), as shown by the equation of the Security Market Line (SML) corresponding to the *i*-th security of a specific market:

$$R_i = R_f + \beta_i(R_M - R_f), \quad (1)$$

where R_M is the expected return of the market and R_f is the risk-free rate, usually, of the Treasury bond.

The CAPM considers the underlying hypothesis of market efficiency by assuming that all (financial) investors behave in the same manner, they have the capacity to diversify their portfolio and have liquidity as well as take and lend money at a fixed rate (R_f). This implies that asset prices should equal the present value of cash flows. Thus, investors and managers have the possibility to reduce their risk (beta) through diversification so that they must not be worried about unsystematic or idiosyncratic risk and consequently they should not add any risk premium for it.

Nonetheless, some researchers (e.g., Bartholdy & Peare, 2005; Fama & French, 2004; Levy, 2010; Rossi, 2016) have found that the beta is not the only factor that explains return. Some other variables like size, market to book (M/B) ratio, price to earnings (*P*/*E*) ratio, and leverage (*L*), among others, have a significant explanatory power on stock returns and have not been individually taken into account by the beta (Bali & Cakici, 2004). Lately, some other factors, like the weather (Hirshleifer & Shumway, 2003), some

seasonal patterns (Bouman & Jacobsen, 2002) or inclusively the hormonal levels (Yuan, Zheng & Zhu, 2006) or the gender (Olsen & Cox, 2001) seem prove the influence of the financial investors behavior. In short, all these researches reveal that the behavior of financial investors in the market goes beyond the accounting numbers and financial and economic strategies.

When decision-makers work with non-quoted firms, that is, venture investors concentrated in specific firms, they do not look towards the market. This is the case of privately held companies (herein after referred as PHC), as for example most family firms (FFs), which are the most common firms around the world.

Despite the existence of these two types of investors (financial and venture), practically all the research related to the risk-return relationship (R-RR) is concentrated on the financial risk investors (from now on, FRIs), what is the same, in listed firms, what leaves an important gap for research.

According to the former paragraphs, we can find two key aspects for the analysis of the R-RR from the perspective of the firm. First, the R-RR should be studied in a different manner according to these two types of investors: FRIs, which behave according to the market and are influenced by some psychological factors; and the economic risk investors (from now on, ERIs) a venture investors which are out of the market but behave in a rational economic manner. Second, the FRIs when making their decisions are based on the expected utility theory (Von Neumann & Morgenstern, 1947) and use the market risk-free rate as a reference point to demand a minimum rate of return; in the other hand, the ERI is a venture that put all the eggs in the same nest and, in making decisions, s/he consider s/he is more high-risky than the FRI, thus taking as a reference point the rate of return the FRI can reach in the market.

The main goal of this paper is twofold. First, after explain the difference between FRI and ERI we give support to the minimum rate of return that an ERI requires when facing non-quoted firms due to the different behavior of FRI and ERI. Then, the consequences of this focus are shown empirically. The paper endures a new model of calculus of the minimum rate of return of the owner-investor, called the Three Component's Model (3CM) of special application in the case of PHCs (Rojo-Ramírez, 2014; Rojo-Ramírez & Martínez-Romero, 2018) which is developed and founded in some different ways including the first proposition of Modigliani and Miller (Modigliani & Miller, 1958, 1963).

This research contributes to the existing literature giving a new perspective in evaluating the minimum rate of return in the context of the strategic risk, opening a landscape that will need more research in the field in the future and from different cultures and countries.

The paper is organized as follows. After this introduction, Section 2 is engaged in explain the difference between FRI and ERI and revise the existing literature about the different proposals to calculate the minimum rate of return required by an ERI. Section 3 gives the logical support to the existence of a specific premium to be applied to an ERI in the context of PHCs and from different points of view, given as a result the so-called 3CM. Section 4 widens the focus from the perspective of Mathematics and supports it in the (Modigliani & Miller, 1958, 1963) first proposition. Section 5 applies the 3CM to a sample of firms quoted in the Spanish market by comparing then with the CAPM and discussing results. Finally, Section 6 summarizes and concludes.

2 Antecedents

Scholars usually recognize that calculate the opportunity cost of capital is one of the most difficult task in financial management (Livingston, 2014) and one of the most

important yardstick in investment decisions and valuation (Habib, 2006) that, additionally, has a strong relation with risk (Palliam, 2005a).

Although most companies are non-quoted, main research about cost of capital have been done on these kind of firms and based on the equilibrium theory of capital markets supported in the mean-variance framework. This theory does not consider the existence of different types of investors, particularly those who invest in PHCs like most FFs. Indeed, this is a major weakness of the aforementioned research.

2.1 Financial vs. economic risk investors

When talking about the risk-return relationship R-RR, two points of view can be adopted: the point of view of the security markets (affecting financial risk investors, FRIs) and the point of view of the firm (affecting economic risk investors, ERIs). Although both approaches are related, the analysis is different, particularly in measuring risk.

FRIs pay special attention to the investment liquidity (Gibson & Mougeot, 2004) and individual assets diversification as well as the quantity and the quality of the available information (Yee, 2006), so they invest in financial assets like, for example, shares or bonds. Inside FRIs, different investors can be found. For example, Olsen (1997) found differences between professional and individual (household) investors; Brennan & Cao (1997) found that foreign investors act in a different manner from domestic investors do; and Grinblatt & Keloharju (2000) differentiate between six different kinds of financial investors at an international level based on a classification system established by the EU.

FRIs usually are professionals or investors who work creating portfolios in order to reduce risk and to guarantee a rate of return above a minimum, known as cost of capital (k). Although some venturing investors (e.g., venture capital) behave in a similar way to FRIs selecting portfolios, nevertheless their level of risk increases to the extent that holding increases too. Cochrane (2005) points out three different causes. First, investing in private equity is less liquid than in quoted assets. Second, the investments usually represent a large proportion of their wealth. Finally, they usually play a role which affects management and could result in a higher measured financial return.

FRIs expect a rate of return largely based on capital gains (as, for example, equity shares in the financial markets) and, because the market price is grounded in expectations, they have large psychosocial components that made prices too volatile due to noise traders (Black, 1986). Most of these changes are psychological, as revealed by several authors that found several factors affecting prices like age (Heaton & Lucas, 2000), gender (Olsen & Cox, 2001), the weather (Hirshleifer & Shumway, 2003), some seasonal patterns (Bouman & Jacobsen, 2002), or inclusively the hormonal levels of the investors (Yuan et al., 2006) that can be expected to be more symptomatic in extreme situations of the economy.

On the other hand, ERIs (e.g., family managers) behave in a different manner to professionals (March & Shapira, 1987). The managers' attitude toward risk does not only depend on their individual personality; it also depends on the context, such as organizational incentives and normative, and the expectation about their jobs (Chari, David, Duru & Zhao, 2019). In the particular case of PHCs, risk appreciations could differ substantially from the outsiders and specially owner-managers (Kotlar, De Massis, Fang & Frattini, 2014).

Further, from an economic point of view, Rojo-Ramírez (2014) suggest that, whilst FRIs basically behave accordingly to portfolio theory (Markowitz, 1952, 1959) and the mean-

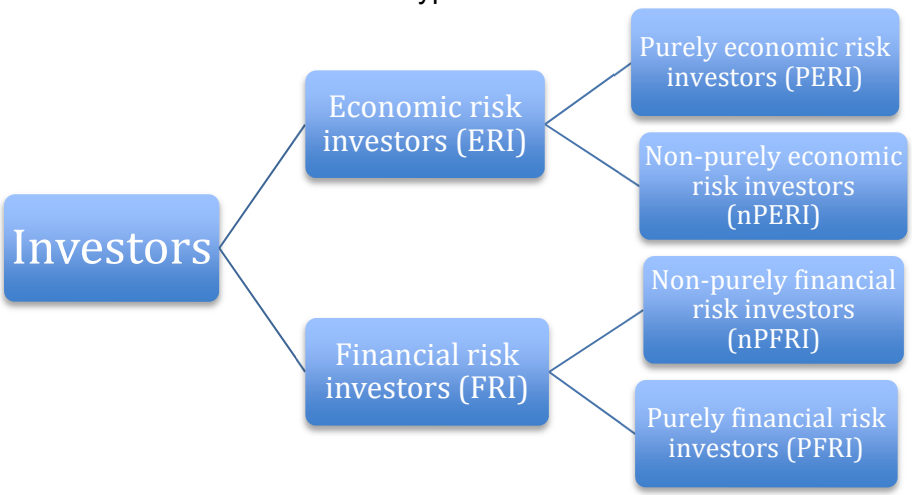
variance framework looking for diversification and liquidity, ERIs are non-diversified venturers whose investments are not enough liquid (Abudy, Benninga, & Shust, 2016). Most of them are owners who stake a substantial part of the firm, as is the case of the owner of a PHC who typically have a large part of their personal net worth invested in a single private firm (Mueller, 2011), being a particular case the family firms (Faccio & Lang, 2002).

ERIs do not create portfolios because they put their personal wealth in a project by assuming a high level of risk. Their ownership stakes usually lacks of market and so it is not liquid. In some cases, as for example a firm whose family members share the control with external investors, we could think that investment is similar to that of FRIs although a purely financial risk investor (PFRI) is not compatible with the family business perspective which takes into account non-financial and long-term goals influencing the minimum rate of return or cost of capital (Martínez-Romero & Rojo-Ramírez, 2017).

The ERIs expect a *k* substantially based on the activity where they operate, embedded in a particular industry. Although different managers and owner-investors have different perceptions of risk, their particular risk is concentrated in the singular activity they develop although influenced by the industry in which they operate and also by the market. For this reason, we can expect that the rate of return of these investors should be smoother than FRIs' rate of return. Further, as ERIs are riskier than FRIs, we also would expect that their *k* will be larger than the corresponding one to FRIs.

Summarizing, to analyze the R-RR it is necessary to define whom investor we are talking about. Chart 1 shows both extremes: PFRI, at the bottom, which is the one that bases his/her behavior on the CAPM, is completely diversified and its investment is totally liquid; purely economic risk investor (PERI), at the top, which is the one that, although bases his/her behavior in an economic rational manner, is totally undiversified and illiquid. Between these two types could be a plethora of possibilities.

Chart 1. Types of investors.



Source: Own elaboration.

2.2 Privately held companies and minimum rate of return

It is well known that CAPM applies when firms are listed but, unfortunately, most companies in all over the world are non-listed. Small and medium enterprises (SMEs) constitute the backbone of the economy of most countries, including EU and US (Abudy et al., 2016). These firms do not quote in any market but their owner-managers have also to take decisions under risk and the minimum required rate of return, *k*, serves as a hurdle rate (McConaughy, 1999) helping them to build growing strategies that support

sustainable competitive advantages and improve firm performance.

As revealed by previous research (Banz, 1981; Brotherson, Eades, Harris, & Higgins, 2013) and by professionals (e.g., Heaton, 1998; London Economics, 2010), in the case of small unlisted firms, it should be necessary to give a risk premium or to add new risk factors because its particular or idiosyncratic risks are not diversifiable and it is reasonable to wholly dispense with the CAPM framework or go beyond the CAPM (Chatterjee, Lubatkin, & Schulze, 1999; Laghi & Di Marcantonio, 2016). This is the especial case of family firms (Martínez-Romero & Rojo-Ramírez, 2017).

Significant efforts have been made in order to apply the CAPM to non-quoted firms since the paper by Hamada, (1972) on unlevered firms' equity scaled by its beta as Livingston, (2014) point out. This mechanism is widely used, particularly within specific industries (CEER Secretariat, 2017; London Economics, 2010). Beside, critics to CAPM also have helped to propose some models based on different components that assume the risk better, particularly after the Fama and French three components model (Fama & French, 1993) as, for example, the five factors model (Fama & French, 2015) or the proposed model for quoted and non-quoted firms of the Laghi & Di Marcantonio, (2016) based in accounting data.

Nonetheless, as we shown above the owners of PHCs have most of their wealth in a single private firm and, as Chatterjee et al., (1999) pointed out, there are three classes of firm-specific risk conditioning its expected returns beyond the macroeconomic uncertainties that have been omitted by the CAPM: Tactical risk, rooted primarily in information asymmetries; Strategic risk, based on imperfections in the resource and output markets, and; Normative risk, entrenched in the forces that underlie institutional norms. This implies that the *k* for a PHC exceeds the cost of capital for listed firms, consistent with the findings reported in the literature on non-diversification (Abudy et al., 2016).

Different solutions have been proposed in the field. For example, Mcnulty, Yeh, Schulze, & Lubatkin, (2002) consider that for practitioners, particularly corporate executives and investment bankers, is better to apply the market-derived capital pricing model (MCPM) a forward looking method base on option market.

Palliam, (2005; 2005a) notes that owners and managers of small businesses do not see their investment as a part of a diversified portfolio but more as a capital project because small business entrepreneurs are creative people that look for niches and they see the risk of failure as just one step closer to success. Thus the CAPM has a limited application being necessary to account for the total risk (systematic and non-systematic or specific) suggesting to apply a five steps Analytical Hierarchical Process (Saaty, 1980) in a similar way as Cotner & Fletcher, (2000) referred to PHC.

In estimating the impact of the non-marketability of the PHC on the cost of equity capital (Abudy et al., 2016) present a method to quantify the premium, either unlevered or levered, considering the influence of taxes based in the framework of (Benninga, Helmantel, & Sarig, 2005). They show the existence of positively relationship between the *k* corresponding to a non-diversified investor and the firm's asset risk and leverage ratio, and to the tax rates in the economy. Finally, Laghi & Di Marcantonio, (2016) develop two models for non-quoted companies that call stand-alone and conjoint approaches that consider firm-specific risk measures as size, operating risk and financial risk.

In the special case of family firms, McConaughy, (1999) point out that exist reasons for not dismissing out of hand the notion of a family effect in the cost of equity capital. Indeed,

Zellweger & Astrachan, (2008) suggest the final value of this firms is over the financial value which is associated with possession attachment and endowment theory.

Adams III et al., (2004) under the hypothesis that family members want to maintain, at minimum, their wealth, go beyond the CAPM and suggest that for privately company owners, the *k* is a theoretical construct that is equal to long-term financial profitability (ROE_{LT}) measured by the ratio:

$$ROE_{LT} = \frac{G_{target}}{1-P_{target}}, \quad (2)$$

where

- G_{target} is the target for the annual growth rate in net (after-tax) profits of the business, and
- P_{target} is the target for the annual proportion of net profit paid out / withdrawn from the business.

In a similar way, de Visscher, Aronoff, & Ward, (2011), propose to determine the *k* with the help of the CAPM adding an illiquidity premium (IP, from 0 to 1) and a family effect (FF, from 0 to 1) according to the expression:

$$k_e = \left[R_f + \beta \cdot \left(R_M - R_f\right)\right] \cdot (1 + IP) \cdot (1 - FE). \quad (3)$$

Rojo-Ramírez, (2014) based on (Rojo-Ramírez, Cruz-Rambaud, & Alonso-Cañadas, 2011) shows that the minimum return of an ERI is larger than that of the FRI and it has three components when considering a firm as a mixed portfolio. Further, (Martínez-Romero & Rojo-Ramírez, 2017) demonstrate that for FFs the minimum rate of return is inversely related to the emotional endowment presented in these firms.

Summarizing, the circumstances influence the calculus of the *k*. For PHCs in which the ERI are embedded and particularly for family PHCs, most researchers and professional agree that it is needed to look for the possibility to apply a model other than CAPM due to the singularity of this kind of firms whose investors behave in a different manner than FRI given the degree of concentration of its wealth and the uniqueness of its investment, which makes it unique.

3 The cost of equity capital in PHCs

3.1 The cost of equity capital for FRIs

In measuring the FRIs' risk it is mainly used the well-known CAPM (Lintner, 1965; Mossin, 1966; Sharpe, 1963) based on the portfolio theory (Markowitz, 1959) and rooted in the expected utility theory (Von Neumann & Morgenstern, 1947) which is supported by the mean variance paradigm. FRIs have the possibility to control the couple return-risk by constructing portfolios in a market in which:

1. Assets' return (*R*) have a normal distribution,
2. There exists the possibility to lend and borrow money at the risk-free rate of interest (R_f), and
3. Investors have all information about the investment and its context.

Thus, they will be able to find a portfolio with a rate of return (ke) greater than the Treasury bills return (R_f) as a function of the market risk Premium ($P_M \coloneqq R_M - R_f$) (e.g., Brealey, Myers, & Allen, 2006; Jarrow, 2018). This is the well-known formula of the CAPM shown in Equation (1) that we rewrite now:

$$R_i = R_f + \frac{S_{e,M}}{\sigma_M^2}(R_M - R_f), \quad (4)$$

where $\frac{S_{e,M}}{\sigma_M^2}$ is the market beta (β_i), being $S_{e,M} = cov(R_e, R_M)$.

As indicated, this formula has been developed under the aforementioned special conditions in the market, the mean-variance framework and the existence of perfect information. Moreover, it has been developed for FRIs who it is assumed that in order to diversify their resources they construct portfolios according to their preferences and circumstances as investors.

Thus, the minimum rate or return applied to FRI (denoted by k_{eFRI}) depends on P_M and the idiosyncratic risk (β_i). To calculate P_M, two different methodologies are often used. The first one is based on historical data as usual when estimating other performance measures (*ex-post* analysis). Implicitly or explicitly, it is assumed that historic results have at least some predictive ability (Sharpe, 1994). The second method is based on surveys to investors by considering that their experience is a good way to predict (*ex-ante* analysis) (Hanlon, Steele & O'Hanlon, 2000). We would admit that P_M is the consequence of one or more of these salient processes regardless of other possible considerations that are not addressed here such as the premium puzzle (Mehra & Prescott, 1985).

CAPM, although criticized (Dayala, 2012; Fama & French, 2004; Jagannathan & Mcgrattan, 1995; Levy, 2010), constitutes the most usual way to measure the idiosyncratic risk of assets in the market. Its instrument is the beta (β), a statistical way ($\beta = \frac{S_{eM}}{\sigma_M^2}$) that considers that the particular security covaries with the market portfolio. It is also supported on historical data.

For professionals, it constitutes an easy way of calculus. Imagine a FRI that likes to know the k_{eFRI}. To do this, s/he looks for the R_f (e.g., 4%) and the P_M (e.g., 4.5%). If its particular β_e is equal to 1, then

$$k_{ePFI} = 4\% + 4.5\% \cdot 1 = 8.5\%.$$

Note that $\beta_e = 1$ means that the investor is perfectly diversified. In this case, we say that s/he behave as a PFRI.

3.2 The cost of equity capital for ERIs

What about an ERI? As explained in Section 2.1, ERIs are non-diversified and their investments are non-liquid. They are venturers that risk all his/her money usually in only one asset (e.g., a firm or a project). Admitting they behave following the expected utility theory (Von Neumann & Morgenstern, 1947) or the prospect theory (Kahneman & Tversky, 1979) that support decision making, it is necessary to accept that their expected minimum rate of return (k_{eERI}) have to be larger than the reached by a FRI guided by the CAPM. That is, the ERI expects a specific premium we will call P_e.

The parameter P_e constitutes the corporate risk associated with the activities that the firm will develop and goes beyond beta and it has sense to the extent that ERIs support more risk being concentrated in the singular activity they develop.

Consider an economic agent who invests all her/his budget in a portfolio whose rate of return is R_P and whose standard deviation is σ_P. Simultaneously, this investor gets the same amount of budget into debt at the risk-free interest rate (R_f) and s/he invests the

total amount on an economic activity whose rate of return is R_e and whose standard deviation is σ_e. Thus, the total rate of return (R_T) obtained by this economic investor will be:

$$R_T = R_f + \beta_e(R_M - R_f) + R_e - R_f. \quad (5)$$

Equation (5) is compatible with the following reasoning:
An ERI takes as a benchmark the FRI to assure a minimum rate of return, but because s/he go into a specific industry where s/he expects to reach a profit due to a demand of the produced good or service greater than the supply according with s/he own risk (Mankiw, 2011) it is expected that the investment return variability behave in the same way as the industry. In this case, we say that the investor is an ERI (see Chart 1) and therefore, it is expected that

$$cov(R_e, R_s) = \sigma_e \cdot \sigma_S$$

and, so beta became a total beta. That is,

$$\beta_T = \frac{cov(R_e, R_S)}{\sigma_S^2} = \frac{\sigma_e}{\sigma_S}.$$

Consequently,

$$R_T = R_f + \beta_e(R_M - R_f) + \frac{\sigma_e}{\sigma_S}(R_S - R_f). \quad (6)$$

being $\beta_e = 1$.

We can conclude that, if the investor is an ERI, the k_{eERI} is equal to R_T, because its investment is singular.

4 Generalizing a specific risk premium for ERIs

In the above Subsection 3.2, we have seen that the minimum rate of return required by an ERI must be greater than the required by a FRI, the difference being $\frac{\sigma_e}{\sigma_S}(R_S - R_f)$. This is logical because an ERI is requiring a financial return (the same as a FRI) and, moreover, s/he is promoting businesses and, consequently, an adding value for society with the additional risks that the economic activity involves.

We can also deduce the expression (6) by using a reasoning based on the hypotheses of Modigliani-Miller (Modigliani & Miller, 1958, 1963) and the results derived from the CAPM. This will allow us to determine the additional minimum rate of return required by an ERI by using the same methodology as a FRI in the context of the CAPM.

Our proposal is based in the different behavior of the ERIs and FRIs shown in Table 1. For an ERI, the reached accounting rate of return (ROE) by the company C is given by the expected before-tax earnings stream (hereinafter, E_{bT}) generated by its equity investment (denoted by E_C), as it is shown in Table 1. On the other hand, the profitability obtained by a FRI (denoted by *R*) is the ratio $P + D$ to E_C (where *P* means the capital gains, and *D* the dividend).

Table 1. Parallelism between the investments carried out by ERIs and FRIs.

	Type of investor	
Elements and magnitudes	**ERI**	**FRI**

Investment	Economic Assets Liabilities	Long T. financial assets Short T. financial assets
Income stream	Earning from Good/service	Capital gains & dividend
Rate of return	$ROE_{bT} = \frac{E_{bT}}{E_C}$	$R = \frac{P + D}{E_C}$
Benchmark	Industry	Market index

Source: Authors.

4.1 Solution for a firm inside an industry

According to the proposition 1 of Modigliani & Miller (1958:266), the rates of return of the shares issued by the companies inside the same industry are perfectly correlated; said in other words, they are proportional between them or, what is the same, they differ at most by a "scale factor". This allows defining an equivalence relation between all firms belonging to an economic cluster in such a way that a given industry is an equivalence class characterized because all its firms have perfectly correlated rates of return.

In summary, by assuming this proposition, all firms inside a given industry will present perfectly correlated rates of return on their respective shares or equity. So, we can use the tools from the CAPM to obtain the specific rate of return of an ERI (R_e) using the Lagrange multipliers method (Rojo-Ramírez et al., 2011):

$$R_e = \left(R_S - R_f\right)\frac{\sigma_e}{\sigma_S}, \tag{7}$$

where, σ_e is the standard deviation of the C's rate of return and R_S and σ_S are, respectively, the rate of return and the standard deviation corresponding to the industry in which the company C is embedded. Thus, taking into account equations (6) and (7), we can conclude that the minimum rate of return of an ERI embedded in an industry will be ($\beta_e = 1$):

$$k_{eERI} = R_f + P_M + \left(R_S - R_f\right)\frac{\sigma_e}{\sigma_S} \tag{8}$$

or, what is the same,

$$k_{eERI} = R_M + \left(R_S - R_f\right)\frac{\sigma_e}{\sigma_S}. \tag{9}$$

However, expressions from (6) to (9) involve calculating σ_S which most times is not known whereby it is necessary to use another benchmark whose data are available. This could be the case of the whole market.

4.2 Solution for a firm inside the whole market

Let C be an arbitrary company embedded in a perfect market and assume that it is composed by n economic assets, denoted by $A_1, A_2, \ldots, A_n$[1]. The ROE corresponding to the *i*-th asset will be ROE_i because the uncontrolled and/or speculative components should be eliminated, due to the absence of arbitrage (Hull, 2008). This means that the profitability obtained by an owner-investor of the company is the weighted average of the accounting rates of return obtained through the investment in each asset. Therefore, the minimum rate of return reached by the company C (denoted by k_{eERI}) will be:

[1] Note that a generic asset A_i implies an investment or a credit of €M_i on a given company C_i.

$$k_{eERI} = \frac{\sum_{i=1}^{n} M_i \cdot ROE_i}{\sum_{i=1}^{n} M_i}, \quad (10)$$

where some ROE_i can be R_f, for example, in case that the *i*-th asset is a loan or credit of €M_i at the risk-free interest rate.

Note that now the value of k_{eERI} is in the same line as (Modigliani & Miller, 1958) when stating: […] *the price of a commodity representing a "bundle" of two other commodities cannot be consistently different from the weighted average of the prices of the two components (the weights being equal to the proportion of the two commodities in the bundle*).

In a competitive economy, all firms inside a given market of this economy sell an identical product. Moreover, each company produces its goods or services at exactly the same price and by using the same production techniques as the other companies inside such market (Estrin & Laidler, 2008; Gravelle & Rees, 2004). Therefore, by the general equilibrium of markets (Pindyck & Rubinfeld, 2018), all coefficients M_i in Equation (10) must be proportional for all firms belonging to the same market inside the economy.

On the other hand, in this competitive economy, a market portfolio would try to represent all sectors involved in this economy, by including the stocks of all productive sectors of this economic system and, moreover, in the same proportion. For example, a technological market index would try to include the presence all technological sectors operating in this economy.

Consequently, the stock market would be perfectly competitive and the composition of the market portfolio should be perfectly correlated with the assets composition of every company belonging to the market. Therefore, by considering the assets of a company C_i as a portfolio composed by a portion (X_1) of risk-free asset (e.g., long-term Treasury bill) and another portion (X_2) of risky assets *P*, one has:

$$k_{eERI} = X_1 R_f + X_2 R_P$$

where R_f is the random variable which represents the return of the risk-free asset or borrowed amount, and R_P is the random variable which represents the return of the portfolio *P* composed by assets in the stock market. Thus, according to the equilibrium theory of capital markets (see, e.g., Sharpe, 1964; Merton, 1987) ($\beta_e = 1$):

$$R_e = R_f + \frac{R_M - R_f}{\sigma_M}\sigma_e, \quad (11)$$

which represents a measure of the rate of return obtained by the total assets of a company (k_{eERI}) according to its risk (σ_e).

Finally, taking into account equations (4) and (5), expression (11) would remain:

$$k_{eERI} = R_M + \frac{R_M - R_f}{\sigma_M}\sigma_e \quad (12)$$

or

$$k_{eERI} = R_f + P_M + P_M \frac{\sigma_e}{\sigma_M}, \quad (13)$$

where $\frac{\sigma_e}{\sigma_M}$ is also known as total beta and has gained some relevance in the professional field (Alonso-Cañadas & Rojo-Ramírez, 2011; Butler & Pinkerton, 2009).

5 Consequences of apply the CAPM vs. 3CM

In order to appreciate the significance of the propose model and its impact in the value of the firm it is applied to the firms listed in the Madrid Stock Exchange General Index (IGBM) for the period 2005-2015. In December 2015 there were 152 firm listed in the Spanish stock Exchange but only 129 firm in the continuous market (*Mercado continuo, CM*).

The data were obtained from two main sources. Market data were from the Madrid stock exchange and its Research Department. The economic and accounting data were from SABI. After exclude those firms without enough data and some extinguished firms or in liquidation rest 97 (86 in the CM).

The minimum rate of return was calculated for each year by both the CAPM (k_{eCAPM}) and the 3CM (k_{eERI}) propose in section 4. To calculate k_{eCAPM}, a four-year market beta was uses (β_e)[2]. The OECD long-term interest rate for Spain was used as a risk free rate (R_f). A 4.5% market risk premium (P_M) for Spanish market was used according to (Garrido & García, 2010) which is similar to some other researches (e.g. Britzelmaier, Kraus, Häberle, Mayer, & Beck, 2013; London Economics, 2010). The same P_M is also apply to calculate the 3CM looking for comparability.

Table 2 shows a summary of the data by industry according to IGBM. If all firms are computed (continuous and Parqué) the minimum difference between the CAPM and the 3CM ranges from 4.7% (for Oil and energy) to 10.5% (for Basic materials, industry and construction). The total difference for the Spanish market (IGBM) is 7.2%. The most risky industry based in the excepted return is the Basic materials, industry and construction sector. According to the 3CM the minimum rate of return the investors require is 17.3%. This sector is followed by Technology and telecommunications sector (16.8%). Indeed, they exist differences in rates of return among sectors that are considerably large according to CAPM (Fraumeni & Jorgenson, 1980) but shorter for the 3CM. Nevertheless, these percentages are under those suggested by professionals (Tatum, 2010).

Table 2. Minimum rate of return according to CAPM and 3CM by industries.

	Continuous		With Parqué($)		Difference(*)		Nº of firms	
IGBM's Industries(#)	ke CAPM	ke 3CM	ke CAPM	ke 3CM	Continuous	With Parqué	Continuous	with Parqué
1	0,075	0,122	0,075	0,122	0,047	0,047	7	7
2	0,069	0,153	0,067	0,173	0,084	0,105	24	27
3	0,056	0,142	0,055	0,141	0,085	0,085	24	26
4	0,072	0,139	0,072	0,139	0,067	0,067	13	13
5	0,076	0,145	0,072	0,130	0,069	0,057	6	9
6	0,109	0,168	0,109	0,168	0,059	0,059	6	6
7	0,060	0,151	0,058	0,143	0,090	0,084	6	9

[2] Data for 2009 have only for IBEX and are from Bloomberg which use the two year beta and weekly period.

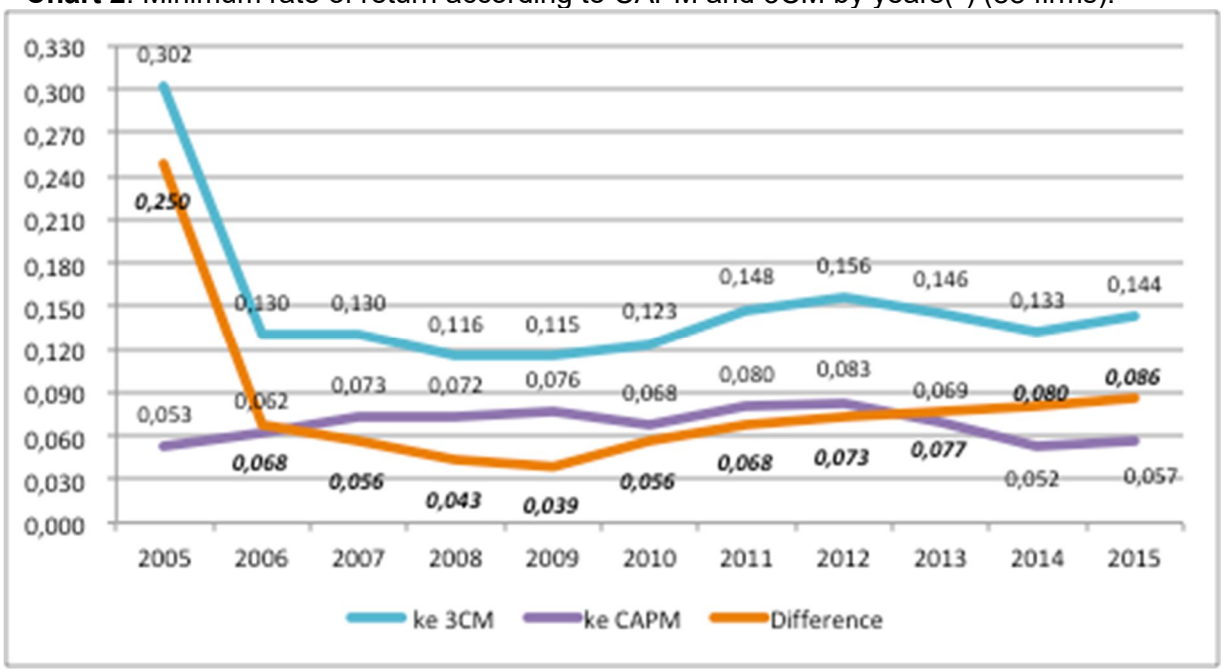
Total general	0,074	0,146	0,073	0,145	0,072	0,072	86	97

(&) Out of the continuous market or in the trading floor
(*) Difference = ke $_{3CM}$ - ke $_{CAPM}$
(#) 1. Oil and energy; 2. Basic materials, industry and construction; 3. Consumers good; 4. Consumers' services; 5. Financial services; 6. Technology and telecommunications; 7. Real estate services

Source: Authors.

Chart 2 shows the minimum rate of return by years. Excluding 2005 that reveal the pre-crisis phenomenon, it can be appreciated that the difference between the two minimum rates of return (k_{eERI} and k_{eCAPM}) have been increased steadily since 2010 reaching the 8.6% in 2015, almost 3 point up in five years. Conversely, it looks that for the crisis period the difference is shorter and reduce basically another 3 point. This trend looks suggest that as economic activity increases, so does the minimum rate of return and vice versa.

Chart 2. Minimum rate of return according to CAPM and 3CM by years(*) (88 firms).



(*) 2009 only includes the betas of the IBEX35 existing in this year that have been obtained from Bloomberg
Source: authors.

This behavior suggest the FRIs in good time are optimistic and reluctant to admit losses instead, the ERI on the other hand, they foresee changes in trends better and adjust their minimum rate of return in order to preserve their projects. This behavior would explain the big change in 2006.

The obtained data have some implications as it is shown in table 3 in which a constant stream of €1,000 equity free cash flow have been discounted using the minimum rate of return, both the k_{eCAPM} and the k_{eERI} displayed in table 2 (with Parqué). As correspond the CAPM value is larger than the 3CM value. The latest columns show the percentage that the difference between the two represents over the CAPM value and the 3CM value. In average the difference value represent a 49.6% and 106.3% respectively of the CAPM and 3CM values.

Table 3. Minimum rate of return according to CAPM and 3CM by industries.

Industry	Value (with Parqué)		Difference in value	% over value ·CAPM	% over value ·3CM
	CAPM	3CM			

1	13.398,60	8.205,85	-5.192,75	38,76%	63,28%
2	14.820,85	5.784,68	-9.036,17	60,97%	156,21%
3	18.027,18	7.109,05	-10.918,14	60,56%	153,58%
4	13.867,94	7.193,77	-6.674,17	48,13%	92,78%
5	13.801,08	7.698,18	-6.102,90	44,22%	79,28%
6	9.153,12	5.945,27	-3.207,84	35,05%	53,96%
7	17.159,12	7.011,49	-10.147,63	59,14%	144,73%
Average	**14.318,27**	**6.992,61**	**-7.325,66**	**49,55%**	**106,26%**

Source: authors.

These data can be interpreted in a twofold sense. First, as Alonso-Cañadas & Rojo-Ramírez, (2012) suggest it can be realized as a discount relate to those firms that are quote, mean that investors undervalue those firms that are not liquid and good enough diversified. In this sense, the final value assigned to the firm should be discount in a 49.6%. This point of view can be supported by the rational fundamental economic principles (Heaton, Lucas, & McDonald, 2010) that consider the market prices are generally the best available measure of economic value, particularly when the liquidity increase (Chordia, Roll, & Subrahmanyam, 2008).

Second, it should be possible to consider that the market is overvalued due to the psychological thinking of financial investors (Shiller, 2003) or because the existence of intangibles (Schauten, Stegink, & de Graaff, 2010) or some other explanations as we said above. This point of view can be supported in a far too volatility of the equity prices than the value based on fundamental cash flows (Bansal & Lundblad, 2002).

6 Conclusion

This article addresses the minimum rate of return required by the owner-investors (k_e) under the expected utility theory (Von Neumann & Morgenstern, 1947). It shows that in order to calculate $k_e$it is needed to consider the existence of different kind of owner-investors according to whether they are financial risk investors or economic risk investors.

The CAPM work satisfactorily good for the FRIs but when working with ERIs it arises the necessity for another more efficient and equally operative alternative that helps investors in their daily work.

It is demonstrated that considering a firm as a bunch of assets that conform a portfolio composed by the risk-free asset and another risky portfolio the k_e of the firm can be measured as the aggregate of the risk-free rate and the market risk premium adjusted by the variability of the risk company with respect to the market. We call this model as the three component's model (3CM) following (Rojo-Ramírez, 2014) based on (Rojo-Ramírez et al., 2011).

The 3CM's empirical application shows that the calculus of the ke by the CAPM model should be scrutinized in order to decide if differences wint the 3CM have to be consider as an overvalue of the firms or the recognition of the greatest liquidity by the listed companies

Contribution
Our study contributes to the existing literature, and in the professional field, by proposing a model to use particularly when firms are non-listed. It demonstrates the proposal made for first time in the Document nº 7 issued by AECA's Commission in 2005.

Limitation.

The proposal about the 3CM in this paper have been developed and proved for the Spanish context requiring be tested in different contexts

References

Abudy, M., Benninga, S., & Shust, E. 2016. The cost of equity for private firms. ***Journal of Corporate Finance***, 37: 431–443.

Adams III, A. F., Manners Jr, G. E., Astrachan, J. H., & Mazzola, P. 2004. The Importance of Integrated Goal Setting: The Application of Cost-of-Capital Concepts to Private Firms. ***Family Business Review***, 17(4): 287–302.

Alonso-Cañadas, J., & Rojo-Ramírez, A. A. 2011. The Discount Rate in Valuing Privately Held Companies. ***Business Valuation Review***, 30(2): 70–81.

Alonso-Cañadas, J., & Rojo-Ramírez, A. A. 2012. Trends in the discount for lack of marketability. ***Valuation Strategies***, 15(4): 22–29.

Bali, T. G., & Cakici, N. 2004. Value at Risk and Expected Stock Returns. ***Financial Analysts Journal***, 60(2): 57–73.

Bansal, R., & Lundblad, C. 2002. Market efficiency, asset returns, and the size of the risk premium in global equity markets. ***Journal of Econometrics***, 109(2): 195–237.

Banz, R. W. 1981. The relationship between return and market value of common stocks. ***Journal of Financial Economics***, 9(1): 3–18.

Bartholdy, J., & Peare, P. 2005. Estimation of expected return: CAPM vs Fama and French. ***International Review of Financial Analysis***, 14: 407–427.

Benninga, S., Helmantel, M., & Sarig, O. 2005. The timing of initial public offerings. ***Journal of Financial Economics***, 75(1): 115–132.

Black, F. 1986. Noise. ***The Journal of Finance***, 41(3): 528–543.

Bouman, S., & Jacobsen, B. 2002. The Halloween Indicator, 'Sell in May and Go Away': Another Puzzle. ***The American Economic Review***, 92(5): 1618–1635.

Brealey, R. A., Myers, S. C., & Allen, F. 2006. ***Principles of Corporate Finance*** (6th ed.). Boston, MA.: McGraw-Hill Interamericana de España S.L.

Brennan, M. J., & Cao, H. H. 1997. International portfolio investment flows. ***The Journal of Finance***, 52(5): 1851–1880.

Britzelmaier, B., Kraus, P., Häberle, M., Mayer, B., & Beck, V. 2013. Cost of capital in SMEs. ***EuroMed Journal of Business***, 8(1): 4–16.

Brotherson, W. T., Eades, K. M., Harris, R. S., & Higgins, R. C. 2013. 'Best Practices' in Estimating the Cost of Capital: An Update. ***Journal of Applied Finance***, 23(1).

Butler, P. J., & Pinkerton, K. 2009. Total Beta: The Missing Piece of the Cost of Capital puzzle. ***Valuation Strategies***, 12(5): 37–39.

CEER Secretariat. 2017. ***CEER Report on Investment Conditions in European Countries***. Brussels, Belgium, UE-Belgium.

Chari, M. D. R., David, P., Duru, A., & Zhao, Y. 2019. Bowman's risk-return paradox: An agency theory perspective. ***Journal of Business Research***, 95: 357–375.

Chatterjee, S., Lubatkin, M. H., & Schulze, W. S. 1999. Toward a strategic theory of risk premium: Moving beyond CAPM. ***The Academy of Management Review***, 24(3): 556–567.

Chordia, T., Roll, R., & Subrahmanyam, A. 2008. Liquidity and market efficiency. ***Journal of Financial Economics***, 87: 249–268.

Cochrane, J. H. 2005. The risk and return of venture capital. ***Journal of Financial Economics***, 75(1): 3–52.

Cotner, J. S., & Fletcher, H. D. 2000. Computing the cost of capital for privately held firms. ***American Business Review***, 18(2): 27–33.

Da, Z., Guo, R.-J., & Jagannathan, R. 2012. CAPM for estimating the cost of equity capital: Interpreting the empirical evidence. ***Journal of Financial Economics***.

Dayala, R. 2012. The Capital Asset Pricing Model: A Fundamental Critique. ***Business Valuation Review***, 31(1): 23–34.

de Visscher, F. M., Aronoff, C. E., & Ward, J. L. 2011. ***Financing Transitions: Managing Capital and Liquidity in the Family Business***. New York: Palgrave McMillan.
Estrin, S., & Laidler, D. 2008. ***Microeconomics*** (5th ed.). New York: Prentice Hall.
Faccio, M., & Lang, L. H. P. 2002. The ultimate ownership of Western European corporations. ***Journal of Financial Economics***, 65(3): 365–395.
Fama, E. F., & French, K. R. 1993. Common risk factors in the returns stocks and bonds. ***Journal of Financial Economics***, 33(1): 3–56.
Fama, E. F., & French, K. R. 2004. The Capital Asset Pricing Model: Theory and Evidence. ***The Journal of Economic Perspectives***, 18(3): 25–46.
Fama, E. F., & French, K. R. 2015. A five-factor asset pricing model. ***Journal of Financial Economics***, 116(1): 1–22.
Fraumeni, B. M., & Jorgenson, D. W. 1980. Rates of Return by Industrial Sector in the United States, 1948-76. ***American Economic Review***, 70(2): 326–330.
Galagedera, D. U. A. 2007. A review of capital asset pricing models. ***Managerial Finance***, 33(10): 821–832.
Garrido, J., & García, D. J. 2010. El largo plazo, un brillante aliado de la inversión en Bolsa. ***Revista Bolsa de Madrid***, 3(185): 20–37.
Gibson, R., & Mougeot, N. 2004. The pricing of systematic liquidity risk: Empirical evidence from the US stock market. ***Journal of Banking & Finance***, 28(1): 157–178.
Gravelle, H., & Rees, R. 2004. ***Microeconomics 3rd ed. Essex, UK*** (3th ed.). London, England: Prentice Hall.
Grinblatt, M., & Keloharju, M. 2000. The investment behavior and performance of various investor types: a study of Finland's unique data set. ***Journal of Financial Economics***, 55(1): 43–67.
Habib, A. 2006. Information Risk and the Cost of Capital: Review of the Empirical Literature. ***Journal of Accounting Literature***, 25: 127–168.
Hamada, R. S. 1972. The effect of the firm's capital structure on the systematic risk of common stocks. ***The Journal of Finance***, 27(2): 435–452.
Hanlon, J. O., Steele, A., & O'Hanlon, J. 2000. Estimating the equity risk premium using accounting fundamentals. ***Journal of Business Finance Accounting***, 27(9&10): 1051–1083.
Heaton, H. B. 1998. Valuing Small Businesses: The Cost of Capital. ***The Appraisal Journal***, 66(1): 11–16.
Heaton, J., & Lucas, D. 2000. Portfolio choice and asset prices: The importance of entrepreneurial risk. ***The Journal of Finance***, 55(3): 1163–1198.
Heaton, J., Lucas, D., & McDonald, R. 2010. Is mark-to-market accounting destabilizing? Analysis and implications for policy. ***Journal of Monetary Economics***, 57(1): 64.
Hirshleifer, D., & Shumway, T. 2003. Good Day Sunshine: Stock Returns and the Weather. ***The Journal of Finance***, 58(3): 1009–1032.
Hull, J. C. 2008. Options, futures and other derivatives. ***Pearson Prentice Hall*** (8th ed.). Boston, MA.: Prentice Hall. https://doi.org/10.1111/0022-1082.00127.
Jagannathan, R., & Mcgrattan, E. R. 1995. The CAPM Debate. ***Federal Reserve Bank of Minneapolis Quarterly Review***, 19(4): 2–17.
Jarrow, R. A. 2018. ***Continuous-Time Asset Pricing Theory: A Martingale-Based Approach***. New York: Springer.
Kahneman, D., & Tversky, A. 1979. Prospect Theory: An Analysis of Decision under Risk. ***Econometrica***, 47(2): 263–291.
Kotlar, J., De Massis, A., Fang, H., & Frattini, F. 2014. Strategic reference points in family firms. ***Small Business Economics***, 43(3): 597–619.
Laghi, E., & Di Marcantonio, M. 2016. Beyond CAPM: estimating the cost of equity considering idiosyncratic risks. ***Quantitative Finance***, 16(8): 1273–1296.
Levy, H. 2010. The CAPM is Alive and Well: A Review and Synthesis. ***European***

Financial Management, 16(1): 43–71.
Lintner, J. 1965. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. ***The Review of Economic and Statistics***, 47(1): 13–37.
Livingston, L. S. 2014. Finding the Discount rate for a Private firm using Public comparables. ***Review of Business & Finance Studies***, 5(1): 37–49.
London Economics. 2010. ***Small business cost of capital***. London, England.
March, J. G., & Shapira, Z. 1987. Managerial perspectives on risk and risk taking. ***Management Science***, 33(11): 1404–1418.
Markowitz, H. M. 1952. Portfolio Selection. ***The Journal of Finance***, 7(1): 77–91.
Markowitz, H. M. 1959. ***Portfolio Selection: Efficient Diversification of Investments***. (JSTOR, Ed.). New York: Yale University Press.
Martínez-Romero, M. J., & Rojo-Ramírez, A. A. 2017. Socioemotional wealth's implications in the calculus of the minimum rate of return required by family businesses' owners. ***Review of Managerial Science***, 11(1): 95–118.
McConaughy, D. L. 1999. Is the Cost of Capital Different for Family Firms? ***Family Business Review***, 12(4): 353–360.
Mcnulty, J. J., Yeh, T. D., Schulze, W. S., & Lubatkin, M. H. 2002. What ' s Your Real Cost of Capital ? ***Harvard Business Review***, 80(10): 5–12.
Mehra, R., & Prescott, E. C. 1985. The equity premium: A puzzle. ***Journal of Monetary Economics***, 15(2): 145–161.
Merton, R. C. 1987. A simple model of capital market equilibrium with incomplete information. ***The Journal of Finance***, 42(3): 483–510.
Modigliani, F., & Miller, M. H. 1958. The Cost of Capital, Corporation Finance and the Theory of Investment. ***American Economic Review***, 48(3): 261–297.
Modigliani, F., & Miller, M. H. 1963. Corporate income taxes and the cost of capital: a correction. ***The American Economic Review***, 433–443.
Mossin, J. 1966. Equilibrium in a capital asset market. ***Econometrica***, 34(4): 768–783.
Mueller, E. 2011. Returns to Private Equity - Idiosyncratic Risk Does Matter! ***Review of Finance***, 15(3): 545–574.
Olsen, R. A. 1997. Investment risk: The experts ' perspective. ***Financial Analysts Journal***, 53(2): 62.
Olsen, R. A., & Cox, C. M. 2001. The Influence of Gender on the Perception and Response to Investment Risk: The Case of Professional Investors. ***Journal of Psychology and Financial Markets***, 2(1): 29–36.
Palliam, R. 2005a. Estimating the cost of capital: considerations for small business. ***The Journal of Risk Finance***, 6(4): 335–340.
Palliam, R. 2005b. Application of a multi-criteria model for determining risk premium. ***The Journal of Risk Finance***, 6(4): 341–348.
Pindyck, R. S., & Rubinfeld, D. L. 2018. ***Microeconomics*** (9th ed.). New Jersey (USA): Pearson.
Rojo-Ramírez, A. A. 2014. Privately held company valuation and cost of capital. ***Journal of Business Valuation and Economic Loss Analysis***, 9(1): 1–22.
Rojo-Ramírez, A. A., Cruz-Rambaud, S., & Alonso-Cañadas, J. 2011. ***A Note on the Operating Return of a Company Under Modigliani-Miller Assumptions***, vol. WP.
Rojo-Ramírez, A. A., & Martínez-Romero, M. J. 2018. Required and obtained equity returns in privately held businesses: the impact of family nature—evidence before and after the global economic crisis. ***Review of Managerial Science***, 12(3): 771–801.
Rossi, M. 2016. The capital asset pricing model: a critical literature review. ***Global Business and Economics Review***, 18(5): 604–617.
Saaty, T. L. 1980. ***The Analytic Hierarchy Process: Planning, Priority Setting, Resource Allocation***. McGraw-Hill.
Schauten, M., Stegink, R., & de Graaff, G. 2010. The discount rate for discounted cash

flow valuations of intangible assets. ***Managerial Finance***, 36(9): 799–811.
Sharpe, W. F. 1963. A simplified model for portfolio analysis. ***Management Science***, 9(2): 227.
Sharpe, W. F. 1964. Capital asset prices: A theory of market equilibriumunder conditions of risk. ***The Journal of Business***, 19(3): 425–442.
Sharpe, W. F. 1994. The Sharpe Ratio. ***The Journal of Portfolio Management***, 21(1): 49–58.
Shiller, R. J. 2003. From efficient markets theory to behavioral finance. ***Journal of Economic Perspectives***, 17(1): 83–104.
Tatum, T. 2010. A New Method for Building a CAPM Discount Rate for Small Businesses Based on SBBI Data. ***Business Appraisal Practice***, III(September 2009): 37–48.
Von Neumann, J., & Morgenstern, O. 1947. ***The Theory of Games and Economic Behavior***, vol. 2. Princeton: Princeton University Press. https://doi.org/10.1177/1468795X06065810.
Yee, K. K. 2006. Earnings quality and the equity risk premium: A benchmark model. ***Contemporary Accounting Research***, 23(3): 833–877.
Yuan, K., Zheng, L., & Zhu, Q. 2006. Are investors moonstruck? Lunar phases and stock returns. ***Journal of Empirical Finance***, 13(1): 1–23.
Zellweger, T. M., & Astrachan, J. H. 2008. On the Emotional Value of Owning a Firm. ***Family Business Review***, 21(4): 347–363.